AN EARLY-ACCEPT MODIFICATION TO THE TEST PLANS
OF MILITARY STANDARD 781C .

BY

DAVID A. BUTLER AND GERALD J. LIEBERMAN

TECHNICAL REPORT NO. 196

17 JUNE 1980

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DEPARTMENT OF OPERATIONS RESEARCH
AND
DEPARTMENT OF STATISTICS
STANFORD UNIVERSITY
STANFORD, CALIFORNIA

AN EARLY-ACCEPT MODIFICATION TO THE TEST PLANS

OF MILITARY STANDARD 781C

By David A. Butler[1] and Gerald J. Lieberman
Oregon State University and Stanford University

1. Introduction and Summary

Military Standard 781C, "Reliability Design Qualification
and Production Acceptance Tests: Exponential Distribution" [2]
covers the requirements for reliability qualification tests
(pre-production) and reliability acceptance tests (production) for
equipment that experiences a distribution of times-to-failure that
is exponential. These requirements include: test conditions,
procedures, and various fixed-length and sequential test plans
with respective accept/reject criteria. This paper is concerned only
with the statistical test plans and the selection and use of these
plans. The Standard contains both fixed-length test plans (Plans
IXC through XVIIC and XIXC through XXIC) and probability-ratio
sequential tests (Plans IC through VIIIC and XVIIIC). Each fixed-
length test plan is characterized by its discrimination ratio (d),
its total test time (T), and its maximum allowable number of failures
to accept (k). If a fixed-length test plan is selected, the total
test duration is essentially set in advance. The only way in which
one of these plans can terminate early is by rejection. For example,
Test Plan XVIIC terminates with a reject decision at the third
failure if this failure occurs before 4.3 units of total test time

have transpired. An accept decision can only be made when 4.3 units of total test time have accrued. Even if the second failure occurs very early, an early reject decision cannot be made; nor can an early-accept decision be made if no failures have occurred, say, by time 4.0. In both of these situations, an early decision would lack statistical validity in failing to guarantee the operating characteristic of the selected plan. Moreover, an early reject decision by the consumer would probably violate contractual agreements with the producer. However, an early-accept decision by the consumer would not be subject to such an objection. Such a decision might seem very desirable to the consumer (government) if testing costs were substantial or if schedule deadlines were near. This paper presents modifications to the fixed-length test plans of MIL-STD-781C which allow early-accept decisions to be made without sacrificing statistical validity. The proposed plans differ from the probability ratio sequential tests in the Standard in that rejection is permitted only after a fixed number of failures have been observed.

2. The Early-Accept Criterion

The early-accept criterion we will consider is as follows. Consider a test plan $\mathscr{P}_k$ with discrimination radio d, total test time T_k, maximum allowable number of failures to accept k ($k \geq 1$), and consumer's risk β. Consider alternative test plans $\mathscr{P}_0$, $\mathscr{P}_1$, ..., $\mathscr{P}_{k-1}$ with the same discrimination ratio, maximum allowable number of failures to accept j ($0 \leq j < k$), and total test

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times $T_j = \frac{1}{2} \cdot \chi^2_{(1-\beta, 2j+2)}$, where $\chi^2_{(1-\beta, 2j+2)}$ is the $(1 - \beta)$ percentile of a chi-squared distribution with $2j+2$ degrees of freedom.[1]/ The producer's risks for test plans $\mathscr{P}_j (0 \leq j \leq k)$ are in decreasing order of j, the test times are in increasing order of j, and the consumer's risks are constant in j (each is β).

The early-accept criterion is as follows: accept at time T_j if at most j failures have occurred up to that time. This alters the original test plan $\mathscr{P}_k$ by allowing early-accept decisions to be made at k time points prior to the total test time T_k. As a result the producer's risk for test plan $\mathscr{P}_k$ is altered. Also, even though each test plan $\mathscr{P}_0$, $\mathscr{P}_1$, ..., $\mathscr{P}_k$ has consumer's risk β, and even though the alternative test plans $\mathscr{P}_0$, $\mathscr{P}_1$, ..., $\mathscr{P}_{k-1}$ were only involved with accept decisions, the consumer's risk of the resulting test is not maintained at β, and indeed, may be significantly greater than β. It is true that if an early-accept decision is made at time T_j, then test plan $\mathscr{P}_j$, had it been selected prior to the start of testing, would have reached the same conclusion. But, by allowing the test results to effectively dictate which test plan is used, the probability calculations involved in determining the consumer's risk are modified by the conditional probabilities which must consequently be incorporated into them.

[1]/This choice is somewhat arbitrary, but is motivated by the use of this rule to guarantee a given consumer's risk for a fixed-length test plan.

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The producer's and consumer's risks for the modified test plans are computed as follows. Let $P_A(\lambda)$ denote the probability of accepting when the true mean time between failures (MTBF) is $1/\lambda$.

$$P_A(\lambda) = \sum_{j=0}^{k} \Pr\{\text{accept at time } T_j\} .$$

Let $A_{(j)} = \Pr\{\text{accept at time } T_j\}$.

Theorem 1. Suppose the true MTBF is $1/\lambda$. Then

$$A_{(j)} = \frac{(\lambda T_j)^j \exp(-\lambda T_j)}{j!} - \sum_{\ell=0}^{j-1} A(\ell) \cdot \frac{[\lambda(T_j - T_\ell)]^{j-\ell} \exp(-\lambda(T_j - T_\ell))}{(j-\ell)!} .$$

Proof. If an accept decision is made at time T_ℓ, then exactly ℓ failures must have occurred up to that time (since if fewer than ℓ failures had occurred, an accept decision would have been made earlier). Thus

$$\Pr\{\text{exactly } j \text{ failures in } [0, T_j]\} = \Pr\left(\bigcup_{\ell=0}^{j} \{\text{accept at time } T_\ell \text{ and } (j-\ell) \text{ failures in } (T_\ell, T_j]\} \right),$$

where $\bigcup_{D}$ represents a union of disjoint events.

$$\frac{(\lambda T_j)^j \exp(-\lambda T_j)}{j!} = \sum_{\ell=0}^{j-1} A(\ell) \cdot \frac{[\lambda(T_j - T_\ell)]^{j-\ell} \exp(-\lambda(T_j - T_\ell))}{(j-\ell)!} + A_{(j)} . \quad \square$$

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The consumer's risk for the early-accept test plan is $P_A(1)$ and the producer's risk is $1 - P_A(1/d)$.

3. Early-Accept Test Plans

It has been proposed that the early-accept criterion be used with the existing parameters of the fixed-length test plans of MIL-STD-781C. The effect of incorporating the early-accept criterion into these fixed-length test plans (without further modification) is shown in Table 1. In all plans except Plan XXIC the consumer's risk is increased and the producer's risk is decreased. (Test Plan XXIC is unchanged since it only accepts when there are no failures.) The changes are substantial; often the consumer's risk is more than doubled and the producer's risk halved. By altering the test time and the maximum number of failures to accept, it is possible to correct for the effect of the early-accept modification and closely match the operating characteristics (at two points) of the standard fixed-length test plans. The corrections for each of the MIL-STD-781C fixed-length test plans are given in Table 2. Accept times for these early-accept test plans are listed in Table 3.

The corrections were computed by defining functions $f_\alpha(T,k)$ as the producer's risk for an early-accept test plan with parameters T and k, and $f_\beta(T,k)$ as the consumer's risk. As T increases f_α increases and f_β decreases, and as k increases f_α decreases and f_β increases. Because of the integer restriction on k, it is

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not always possible to design a test plan to achieve specified
values of α, β exactly. However, an algorithm which will
determine an approximate solution can be constructed. The algorithm
from which Table 2 is derived first fixes k and uses a quasi-Newton
method to determine a value of T which will achieve the desired
α-value. The process in then repeated, varying k in accordance
with a bisection search, to determine a k-value for which β is
also close to the desired level. Some additional checks to reduce
the calculations are also incorporated. It should be noted that
the test plans of Table 2 are designed to have α and β levels
close to the nominal values of the standard test plans, not the
actual values. (See Tables II and C-1 in [2]).

4. Performance of the Early-Accept Test Plans

Table 2 shows that the maximum test times for the early-accept
test plans are substantially increased from the standard test times.
However, the expected test times for the early-accept plans are
much smaller than the maximum times, and compare quite favorably
to the (fixed) test times for the standard plans.$\frac{1}{}$ Graphs of expected
test duration versus true MTBF for the early-accept test plans appear
in Figures 1-12. If estimation of the true MTBF is not necessary, a
regular fixed-length test can be terminated "early" when rejection is
the appropriate action. Therefore, each figure also graphs the expected
test duration versus the true MTBF for this procedure. It is not

1/ The expected test times for Early-Accept Plans IXC and XC exceed those
for the corresponding standard plans for a considerable range of the
true MTBF. The reason for this is that these two early-accept plans
have producer's and consumer's risks substantially closer to the nominal
values than do the standard plans.

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surprising that the early-accept test plans generally have smaller expected test durations.

The expected test durations are computed as follows. Let τ be the (random) test duration.

$$E_\lambda[\tau] = E\left[\tau \cdot \sum_{j=0}^{k} \left(I_{\{\text{accept at time } T_j\}} + I_{\{\text{reject in } (T_{j-1}, T_j]\}}\right)\right]$$

$$= \sum_{j=0}^{k} T_j \cdot A_{(j)} + \sum_{j=0}^{k} E_\lambda\left[\tau \cdot I_{\{\text{reject in } (T_{j-1}, T_j]\}}\right], \qquad (1)$$

where I_B denotes the indicator function of the event B, i.e., I_B equals 1 if the event occurs, 0 otherwise. To compute the terms in the second summation in (1), note that for the plan reject in $(T_{j-1}, T_j]$, r failures $(j \leq r \leq k)$ must occur prior to T_{j-1}, and $k + 1 - r$ failures must occur in $(T_{j-1}, T_j]$. Thus,

$$E_\lambda\left[\tau \cdot I_{\{\text{reject in } (T_{j-1}, T_j]\}}\right] = \sum_{r=j}^{k} Q(j-1, r) \cdot \int_0^{T_j - T_{j-1}} (T_{j-1} + z) f(z) \, dz, \qquad (2)$$

where $f(z)$ is a gamma density with parameters λ, $k + 1 - r$, and $Q(j, r) = \Pr\{\text{do not accept or reject at or before time } T_j, \text{ and } r$ failures in $[0, T_j]\}$, $(j < r)$.

Theorem 2. For $j < r \leq k$

$$Q(j, r) = \frac{(\lambda T_j)^r \exp(-\lambda T_j)}{r!} - \sum_{\ell=0}^{j-1} A(\ell) \cdot \frac{[\lambda(T_j - T_\ell)]^{r-\ell} \exp(-\lambda(T_j - T_\ell))}{(r-\ell)!}.$$

<u>Proof</u>. For $j < r \leq k$,

$$\text{Pr(exactly r failures in } [0,T_j]) = \text{Pr}\left(\bigcup_{\ell=0}^{j} \text{((accept at time } T_\ell \text{ (necessarily with } \ell \text{ failures) and } (r-\ell) \text{ failures in } (T_\ell, T_j]]\right) + Q_{(j,r)}.$$

$$\frac{(\lambda T_j)^r \exp(-\lambda T_j)}{r!} = \sum_{\ell=0}^{j-1} \left(A(\ell) \cdot \frac{[\lambda(T_j - T_\ell)]^{r-\ell} \exp(-\lambda(T_j - T_\ell))}{(r-\ell)!} \right) + Q(j,r) . \quad \square$$

All that remains is to compute the integral in (2). This integral can be expressed in terms of the incomplete gamma distribution and evaluated by standard computer subroutines [1].

$$\int_0^{T_j - T_{j-1}} (T_{j-1} + z)f(z)dz = \int_0^{T_j - T_{j-1}} (T_{j-1} + z) \frac{\lambda e^{-\lambda z}(\lambda z)^{k-r}}{\Gamma(k-r+1)} dz$$

$$= \int_0^{\lambda(T_j - T_{j-1})} \frac{(T_{j-1} + x/\lambda)e^{-x}(x)^{k-r}}{\Gamma(k-r+1)} dx$$

Let $I(x,p)$ be the incomplete gamma distribution, that is

$$I(x,p) = \int_0^x \frac{e^{-t}t^{p-1}}{\Gamma(p)} dt.$$

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Then

$$\int_0^{T_j - T_{j-1}} (T_{j-1} + z) f(z) dz = T_{j-1} \cdot I(\lambda[T_j - T_{j-1}], k-r+1) + \frac{\Gamma(k-r+2)}{\lambda \cdot \Gamma(k-r+1)} \int_0^{\lambda(T_j - T_{j-1})} \frac{e^{-x} x^{k-r+1}}{\Gamma(k-r+2)} dx$$

$$= T_{j-1} \cdot I(\lambda[T_j - T_{j-1}], k-r+1) + \frac{k-r+1}{\lambda} I(\lambda[T_j - T_{j-1}], k-r+2).$$

5. Conclusions

From an operating characteristic curve point of view, it does not make much difference whether a fixed-length test plan, a probability-ratio sequential test plan (truncated) or an early-accept plan is chosen, provided each is designed to have the same operating characteristics. Generally, the ordering of the expected test duration is smallest for the probability ratio sequential test plan, followed by the early-accept plan, and largest for the fixed-length plan. The advantage of the early-accept plan, over the probability ratio sequential test plan, is purely psychological. The producer never has a lot rejected early, and early decisions occur only with the desirable outcome (from the producer's point of view) of acceptance. Such an advantage cannot be discounted.

References

[1] IMSL Library Reference Manual, Volume 2, International
Mathematical and Statistical Libraries, Inc., Houston, Texas,
7th Edition, February 1979.

[2] Military Standard 781C, "Reliability Design Qualification and
Production Acceptance Tests: Exponential Distribution", U.S.
Department of Defense, 21 October 1977, AMSC Number 22333.

Acknowledgment

TABLE 1 – Changes in Producer's and Consumer's Risks Resulting from Incorporating Early-Accept Criterion into MIL-STD-781C Test Plans

Test Plan	Discrimination Ratio	Without Early-Accept Option[*]		With Early-Accept Option[†]	
		Producer's Risk (%)	Consumer's Risk (%)	Producer's Risk (%)	Consumer's Risk (%)
IXC	1.5	12.0	9.9	4.9	38.1
XC	1.5	10.9	21.4	3.5	58.8
XIC	1.5	17.8	22.1	6.8	56.4
XIIC	2.0	9.6	10.6	4.7	31.8
XIIIC	2.0	9.8	20.9	4.4	48.4
XIVC	2.0	19.9	21.0	11.3	42.8
XVC	3.0	9.4	9.9	5.9	23.1
XVIC	3.0	10.9	21.3	6.8	38.4
XVIIC	3.0	17.5	19.7	12.5	32.6
(High Risk Plans)					
XIXC	1.5	28.8	31.3	14.0	59.5
XXC	2.0	28.8	28.5	19.4	44.6
XXIC	3.0	30.7	33.3	30.7	33.3

[*] Taken from Tables II and III of MIL-STD-781C and is for the test plan without early-accept modification.

[†] True risk when the early-accept criterion is incorporated.

TABLE 2 — Specifications of Standard and Early-Accept Test Plans

Test Plan	Discrimination Ratio	MIL-STD-781C Test Plans**		Test Time*	No. of Failures to Reject	Producer's Risk for Corrected Plan (Z)†	Consumer's Risk for Corrected Plan (Z)†
		Test Time*	No. of Failures to Reject				
IXC	1.5	45.0	≥ 37	72.2	≥ 55	10.2	10.0
XC	1.5	29.9	≥ 26	51.7	≥ 40	10.1	19.8
XIC	1.5	21.1	≥ 18	32.6	≥ 24	20.1	20.4
XIIC	2.0	18.8	≥ 14	26.0	≥ 17	10.4	10.3
XIIIC	2.0	12.4	≥ 10	19.1	≥ 13	9.9	19.2
XIVC	2.0	7.8	≥ 6	12.6	≥ 8	20.0	18.3
XVC	3.0	9.3	≥ 6	12.8	≥ 7	10.0	8.4
XVIC	3.0	5.4	≥ 4	8.3	≥ 5	10.2	18.7
XVIIC	3.0	4.3	≥ 3	5.2	≥ 3	19.7	19.2
High Risk Plans							
XIXC	1.5	8.0	≥ 7	12.6	≥ 9	29.6	30.8
XXC	2.0	3.7	≥ 3	4.5	≥ 3	29.9	29.1
XXIC	3.0	1.1	≥ 1	1.1	≥ 1	30.7	33.3

*In multiples of θ_1.

**From Tables II and III in MIL-STD-781C.

†Corrected for use with early-accept criterion to achieve true producer's and consumer's risks close to nominal levels as given in Table C-1 of MIL-STD-781C.

TABLE 3 - Accept Times of Early-Accept Test Plans

Test Plan	Accept Times*				
IXC	$T_0 = 4.2$	$T_1 = 6.1$	$T_2 = 7.9$	$T_3 = 9.4$	$T_4 = 11.0$
	$T_5 = 12.4$	$T_6 = 13.9$	$T_7 = 15.3$	$T_8 = 16.6$	$T_9 = 18.0$
	$T_{10} = 19.3$	$T_{11} = 20.7$	$T_{12} = 22.0$	$T_{13} = 23.3$	$T_{14} = 24.5$
	$T_{15} = 25.8$	$T_{16} = 27.1$	$T_{17} = 28.3$	$T_{18} = 29.6$	$T_{19} = 30.8$
	$T_{20} = 32.1$	$T_{21} = 33.3$	$T_{22} = 34.5$	$T_{23} = 35.8$	$T_{24} = 37.0$
	$T_{25} = 38.2$	$T_{26} = 39.4$	$T_{27} = 40.6$	$T_{28} = 41.8$	$T_{29} = 43.0$
	$T_{30} = 44.2$	$T_{31} = 45.4$	$T_{32} = 46.6$	$T_{33} = 47.8$	$T_{34} = 49.0$
	$T_{35} = 50.1$	$T_{36} = 51.3$	$T_{37} = 52.5$	$T_{38} = 53.7$	$T_{39} = 54.8$
	$T_{40} = 56.0$	$T_{41} = 57.2$	$T_{42} = 58.3$	$T_{43} = 59.5$	$T_{44} = 60.7$
	$T_{45} = 61.8$	$T_{46} = 63.0$	$T_{47} = 64.1$	$T_{48} = 65.3$	$T_{49} = 66.5$
	$T_{50} = 67.6$	$T_{51} = 68.8$	$T_{52} = 69.9$	$T_{53} = 71.1$	$T_{54} = 72.2$
XC	$T_0 = 3.2$	$T_1 = 5.0$	$T_2 = 6.6$	$T_3 = 8.1$	$T_4 = 9.5$
	$T_5 = 10.9$	$T_6 = 12.2$	$T_7 = 13.6$	$T_8 = 14.9$	$T_9 = 16.1$
	$T_{10} = 17.4$	$T_{11} = 18.7$	$T_{12} = 19.9$	$T_{13} = 21.2$	$T_{14} = 22.4$
	$T_{15} = 23.6$	$T_{16} = 24.8$	$T_{17} = 26.1$	$T_{18} = 27.3$	$T_{19} = 28.4$

Cont'd.

*Accept at time T_j if j failures have occurred to that time.

TABLE 3 (Cont'd)

Test Plan	Accept Times*
XC (Cont'd)	$T_{20} = 29.6$ $T_{21} = 30.8$ $T_{22} = 32.0$ $T_{23} = 33.2$ $T_{24} = 34.4$
	$T_{25} = 35.6$ $T_{26} = 36.7$ $T_{27} = 37.9$ $T_{28} = 39.1$ $T_{29} = 40.2$
	$T_{30} = 41.4$ $T_{31} = 42.5$ $T_{32} = 43.7$ $T_{33} = 44.8$ $T_{34} = 46.0$
	$T_{35} = 47.1$ $T_{36} = 48.3$ $T_{37} = 49.4$ $T_{38} = 50.6$ $T_{39} = 51.7$
XIC	$T_0 = 3.0$ $T_1 = 4.8$ $T_2 = 6.3$ $T_3 = 7.8$ $T_4 = 9.2$
	$T_5 = 10.5$ $T_6 = 11.9$ $T_7 = 13.2$ $T_8 = 14.4$ $T_9 = 15.7$
	$T_{10} = 17.0$ $T_{11} = 18.2$ $T_{12} = 19.5$ $T_{13} = 20.7$ $T_{14} = 21.9$
	$T_{15} = 23.1$ $T_{16} = 24.3$ $T_{17} = 25.5$ $T_{18} = 26.7$ $T_{19} = 27.9$
	$T_{20} = 29.1$ $T_{21} = 30.3$ $T_{22} = 31.4$ $T_{23} = 32.6$
XIIC	$T_0 = 3.7$ $T_1 = 5.6$ $T_2 = 7.2$ $T_3 = 8.8$ $T_4 = 10.3$
	$T_5 = 11.7$ $T_6 = 13.1$ $T_7 = 14.4$ $T_8 = 15.8$ $T_9 = 17.1$
	$T_{10} = 18.4$ $T_{11} = 19.7$ $T_{12} = 21.0$ $T_{13} = 22.3$ $T_{14} = 23.5$
	$T_{15} = 24.8$ $T_{16} = 26.0$
XIIIC	$T_0 = 2.8$ $T_1 = 4.6$ $T_2 = 6.1$ $T_3 = 7.5$ $T_4 = 8.9$
	$T_5 = 10.3$ $T_6 = 11.6$ $T_7 = 12.9$ $T_8 = 14.1$ $T_9 = 15.4$
	$T_{10} = 16.6$ $T_{11} = 17.9$ $T_{12} = 19.1$

*Accept at time T_j if j failures have occurred to that time.

TABLE 3 (Cont'd)

Test Plan	Accept Times*				
XIVC	$T_0 = 2.7$	$T_1 = 4.4$	$T_2 = 5.9$	$T_3 = 7.3$	$T_4 = 8.7$
	$T_5 = 10.0$	$T_6 = 11.3$	$T_7 = 12.6$		
XVC	$T_0 = 3.5$	$T_1 = 5.4$	$T_2 = 7.0$	$T_3 = 8.6$	$T_4 = 10.0$
	$T_5 = 11.4$	$T_6 = 12.8$			
XVIC	$T_0 = 2.5$	$T_1 = 4.1$	$T_2 = 5.6$	$T_3 = 7.0$	$T_4 = 8.3$
XVIIC	$T_0 = 2.2$	$T_1 = 3.8$	$T_2 = 5.2$		
XIXC	$T_0 = 2.1$	$T_1 = 3.7$	$T_2 = 5.1$	$T_3 = 6.4$	$T_4 = 7.7$
	$T_5 = 8.9$	$T_6 = 10.2$	$T_7 = 11.4$	$T_8 = 12.6$	
XXC	$T_0 = 1.8$	$T_1 = 3.2$	$T_2 = 4.5$		
XXIC	$T_0 = 1.1$				

*Accept at time T_j if j failures have occurred to that time.

TEST PLAN IXC

Figure 1

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Figure 2

17

TEST PLAN XIC

Figure 3

Figure 4
19

TEST PLAN XIIIC

Figure 5

20

TEST PLAN XIVC

Figure 6

21

TEST PLAN XVC

Figure 7
22

TEST PLAN XVIC

Figure 8

23
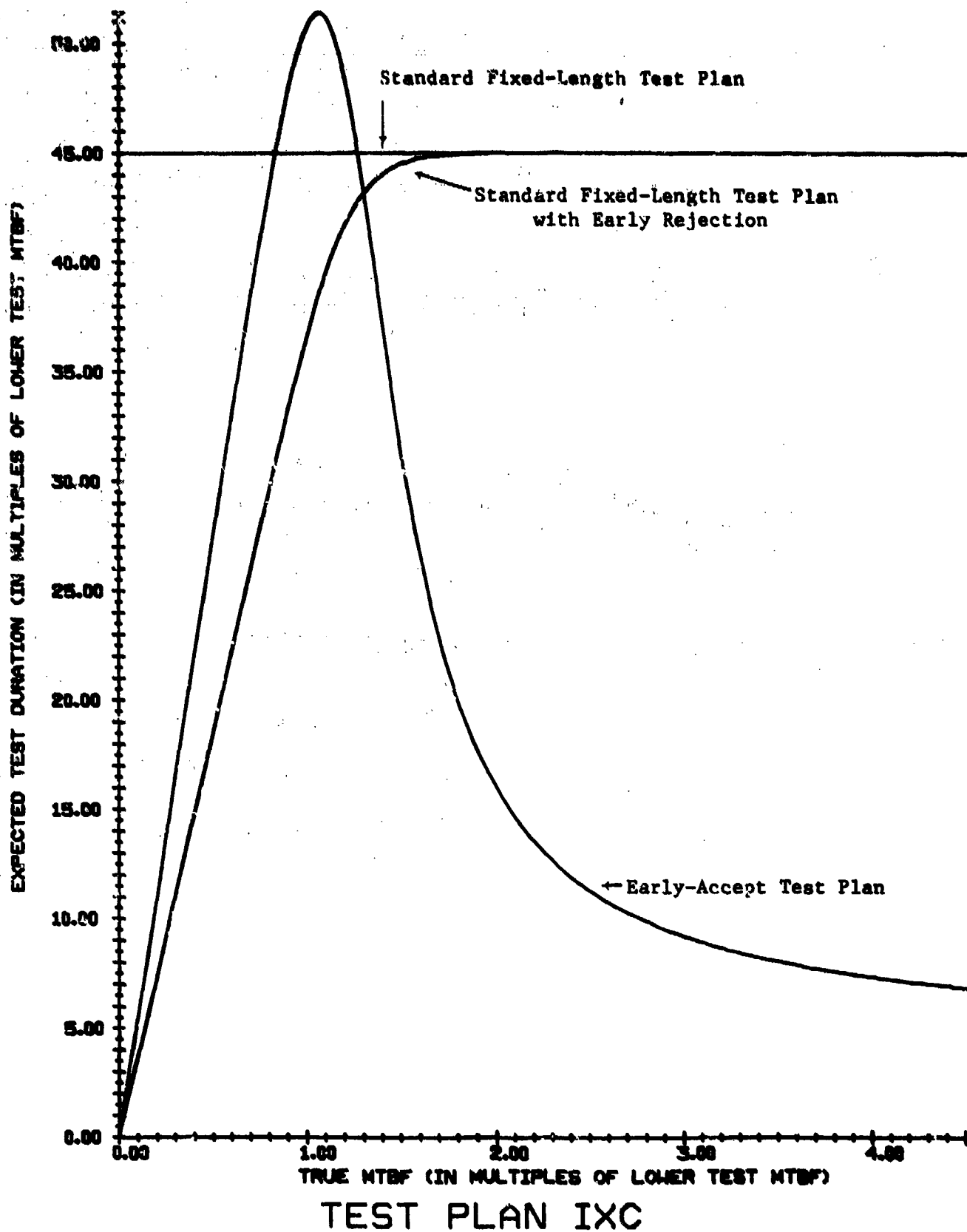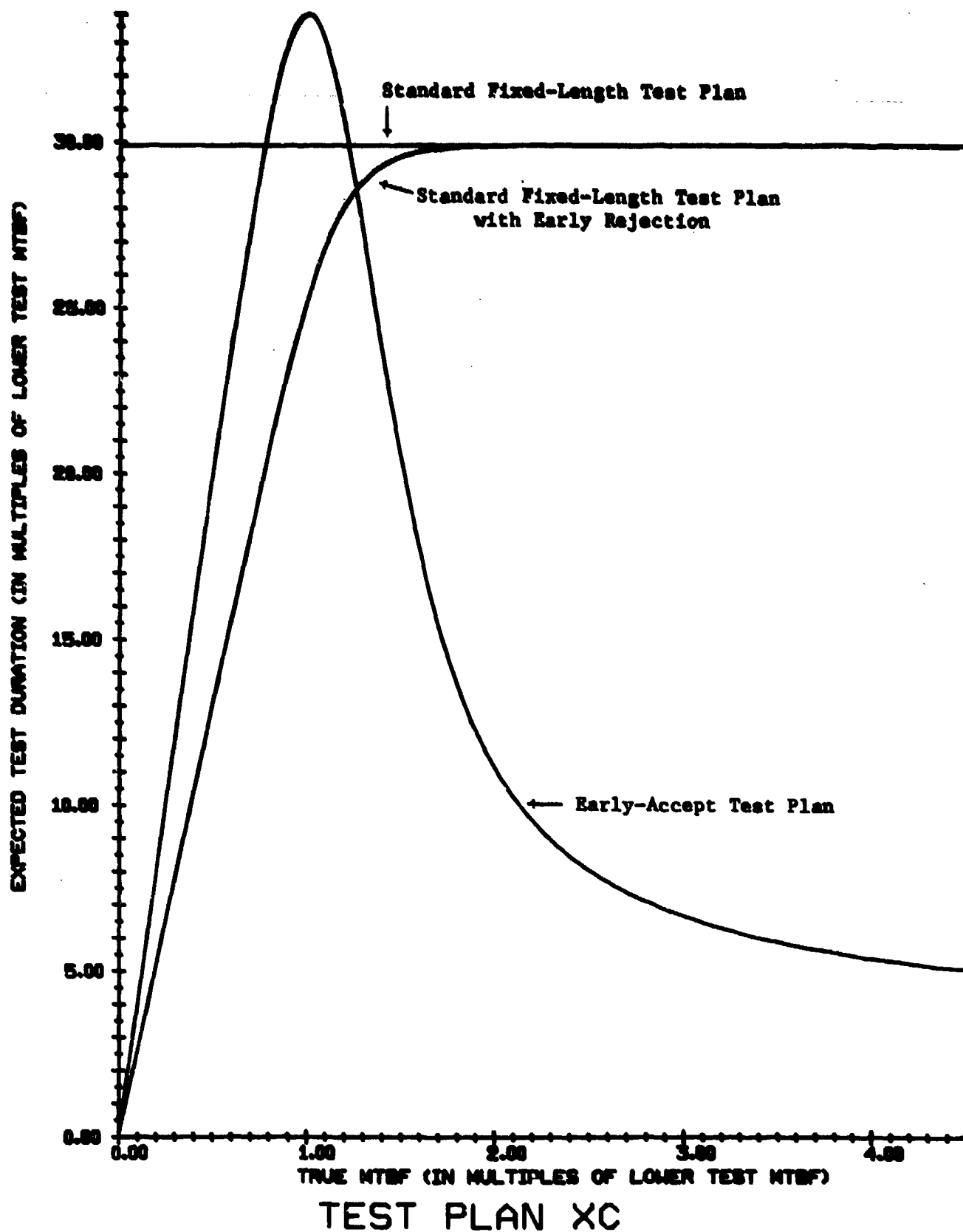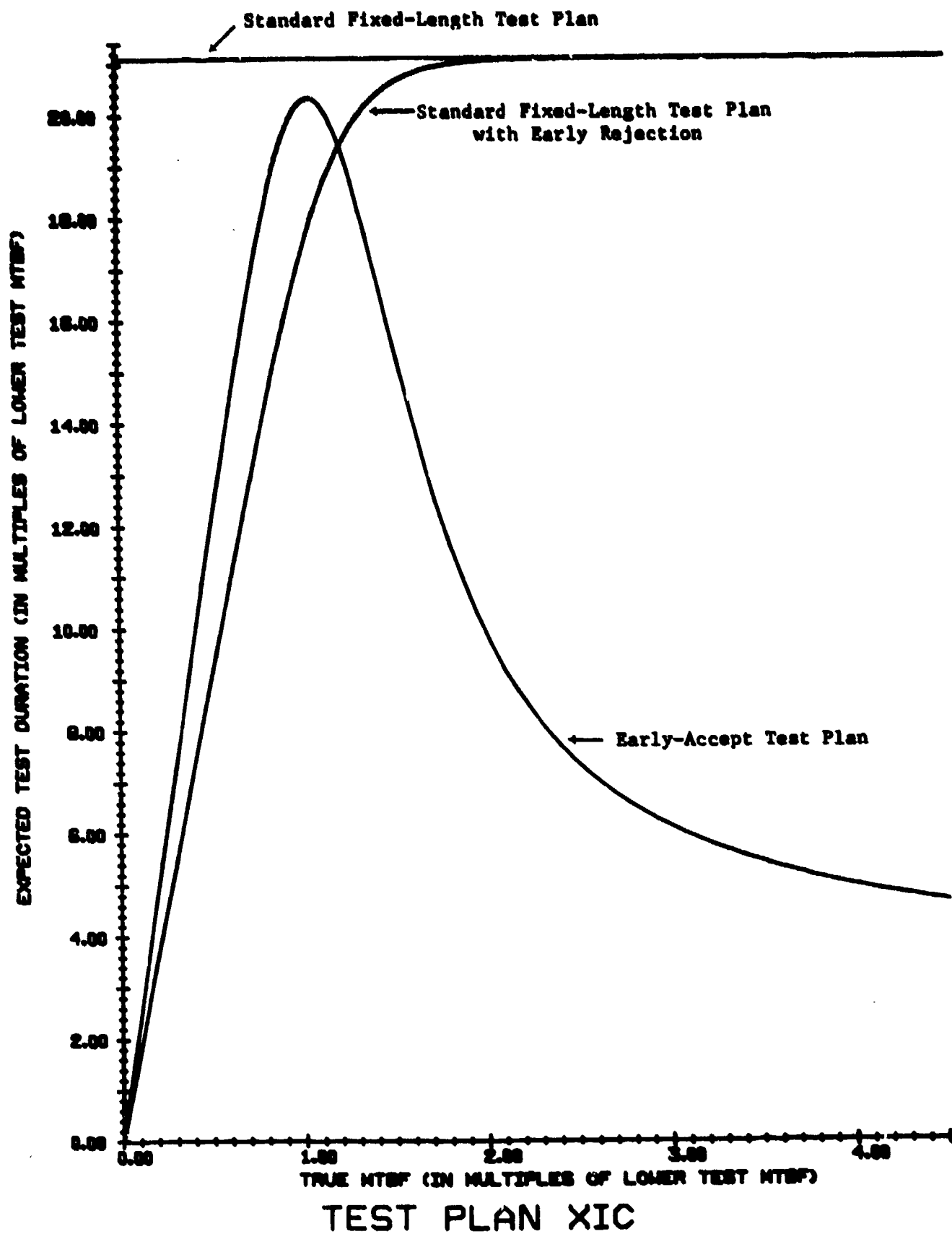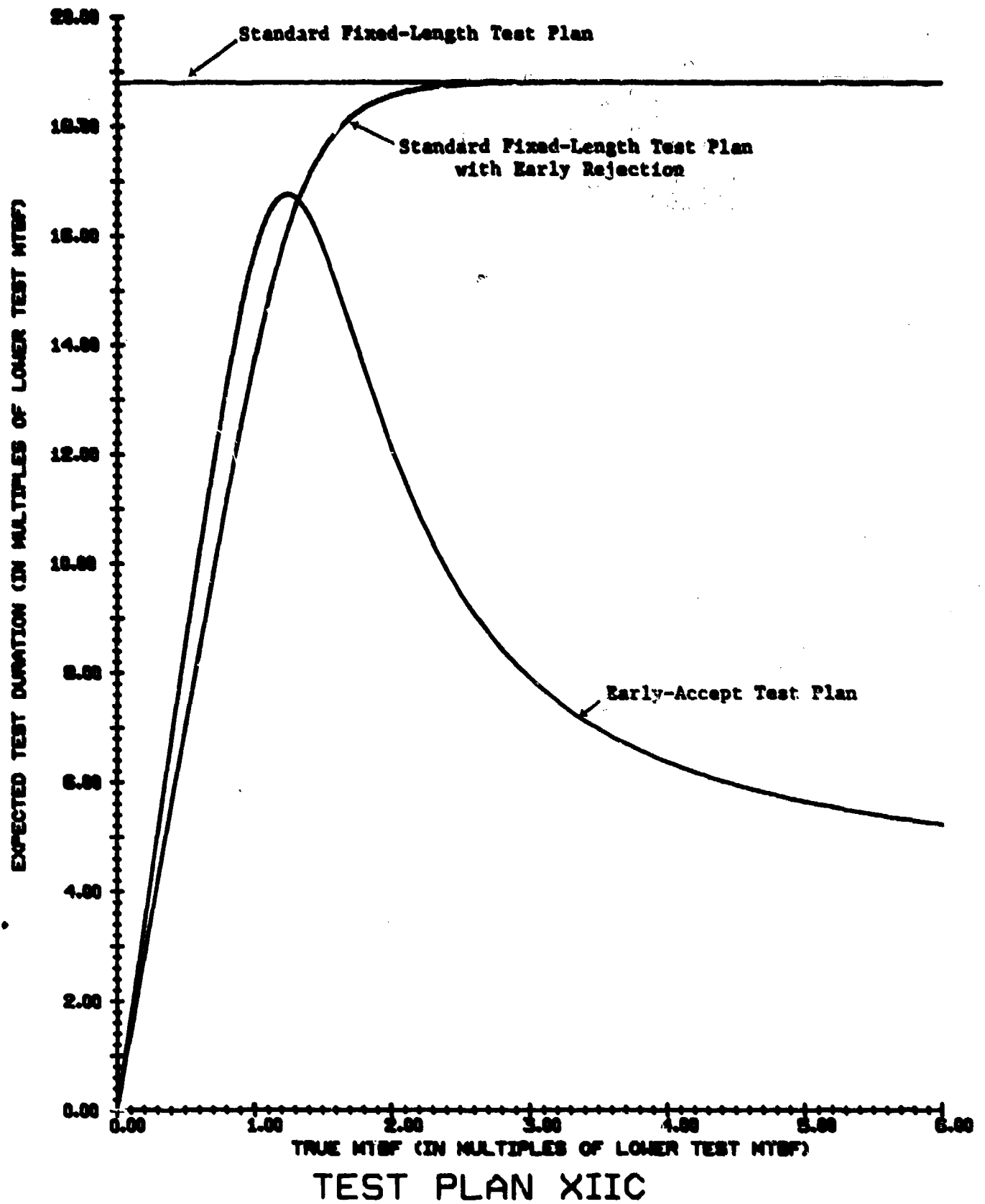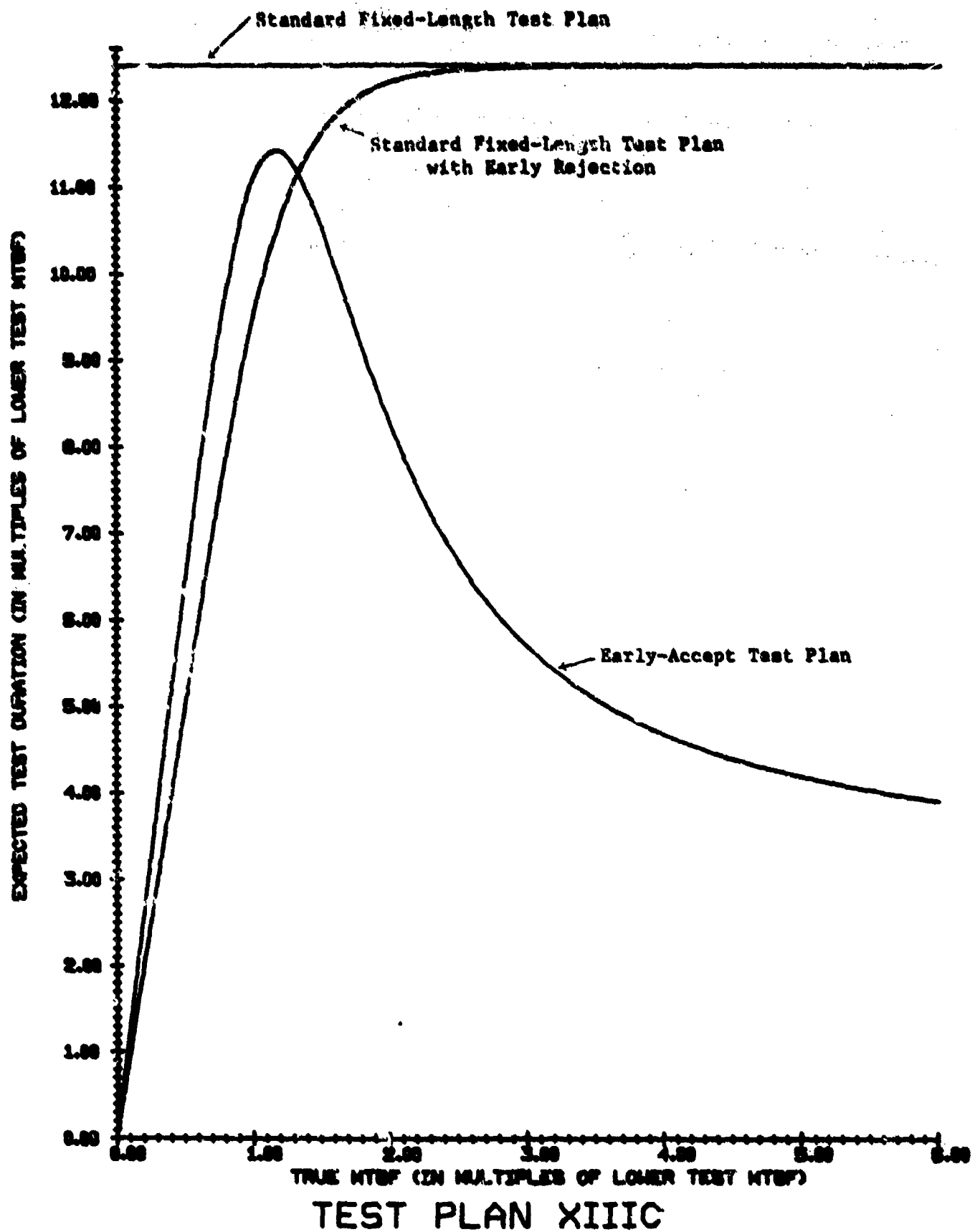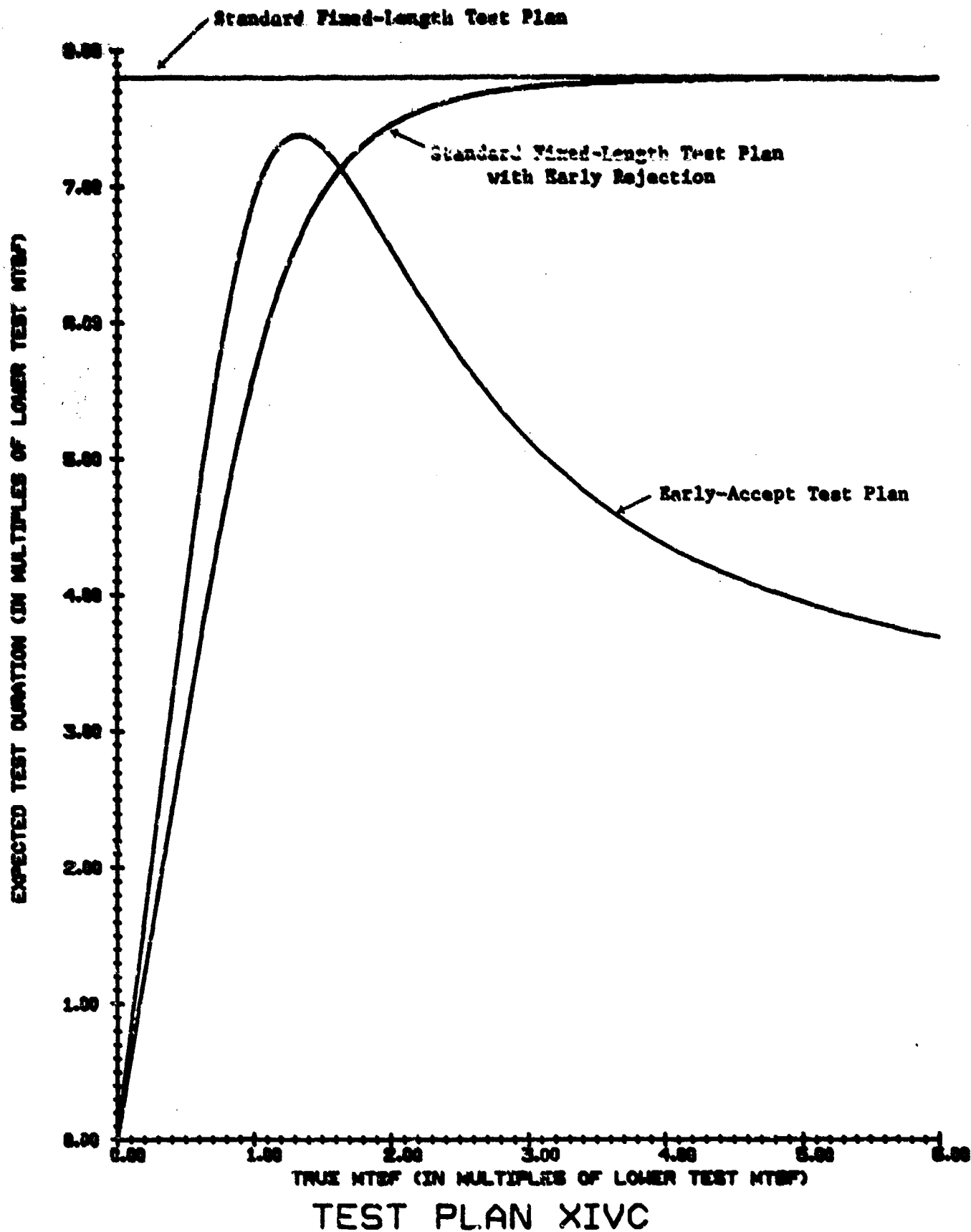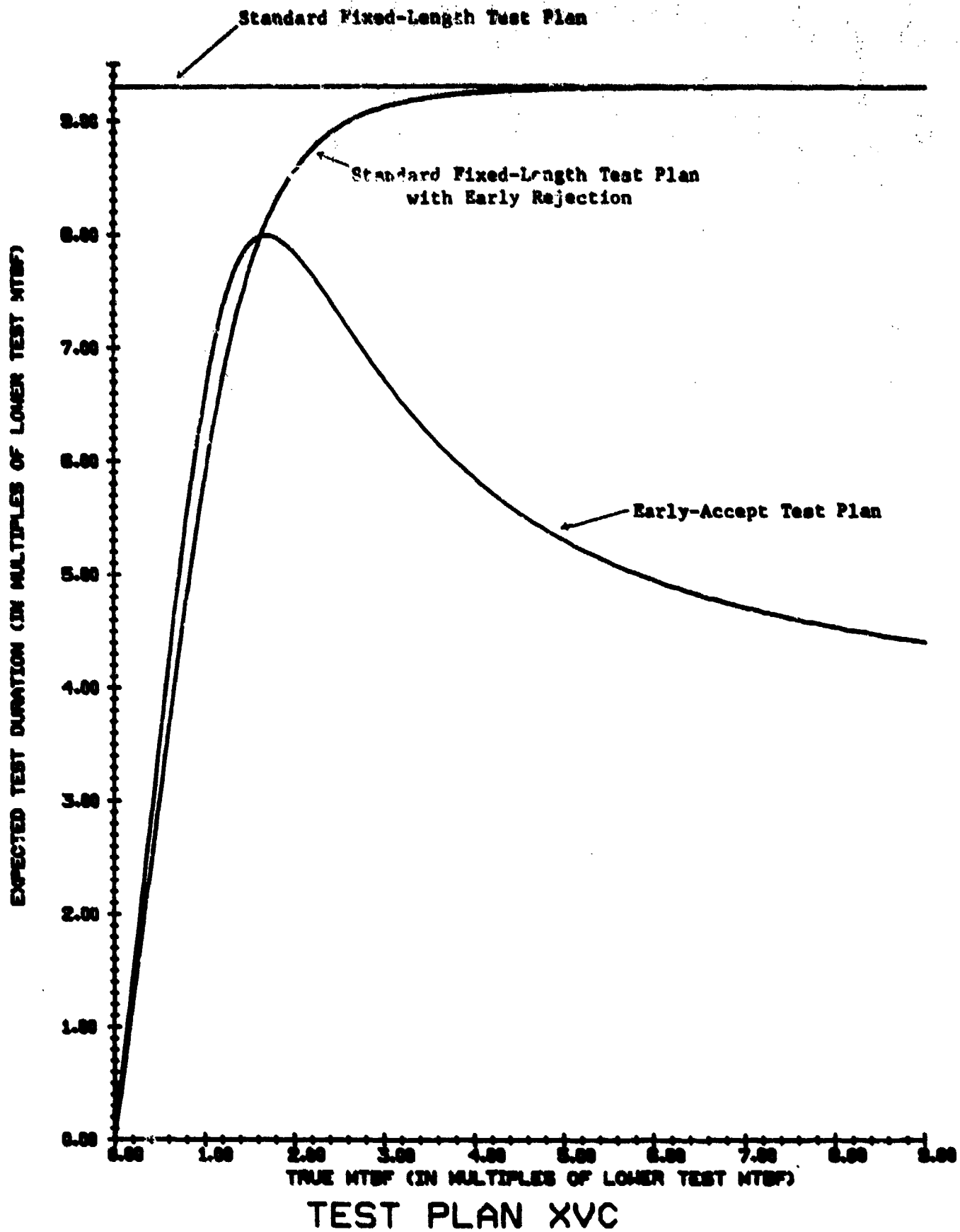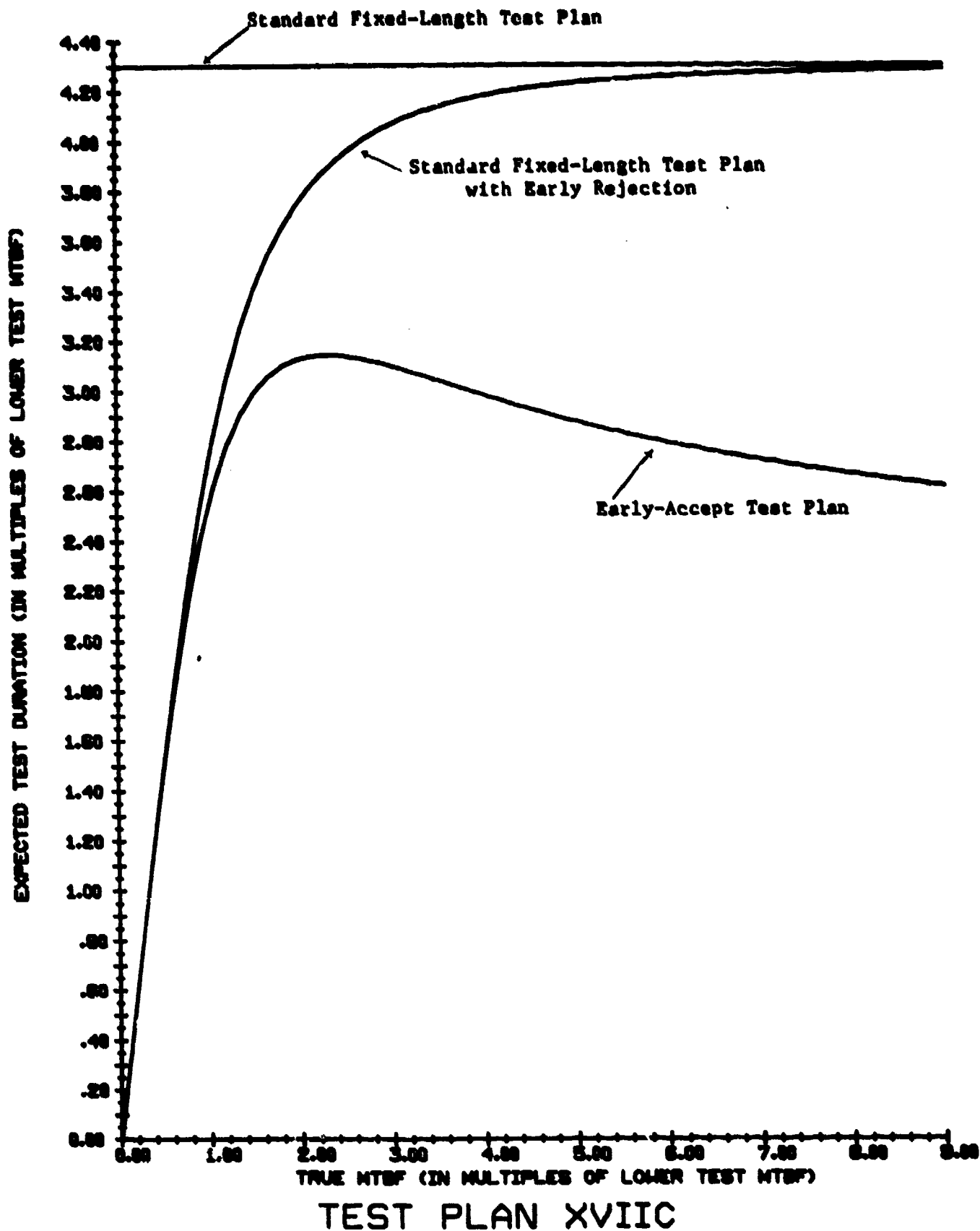
TEST PLAN XVIIC
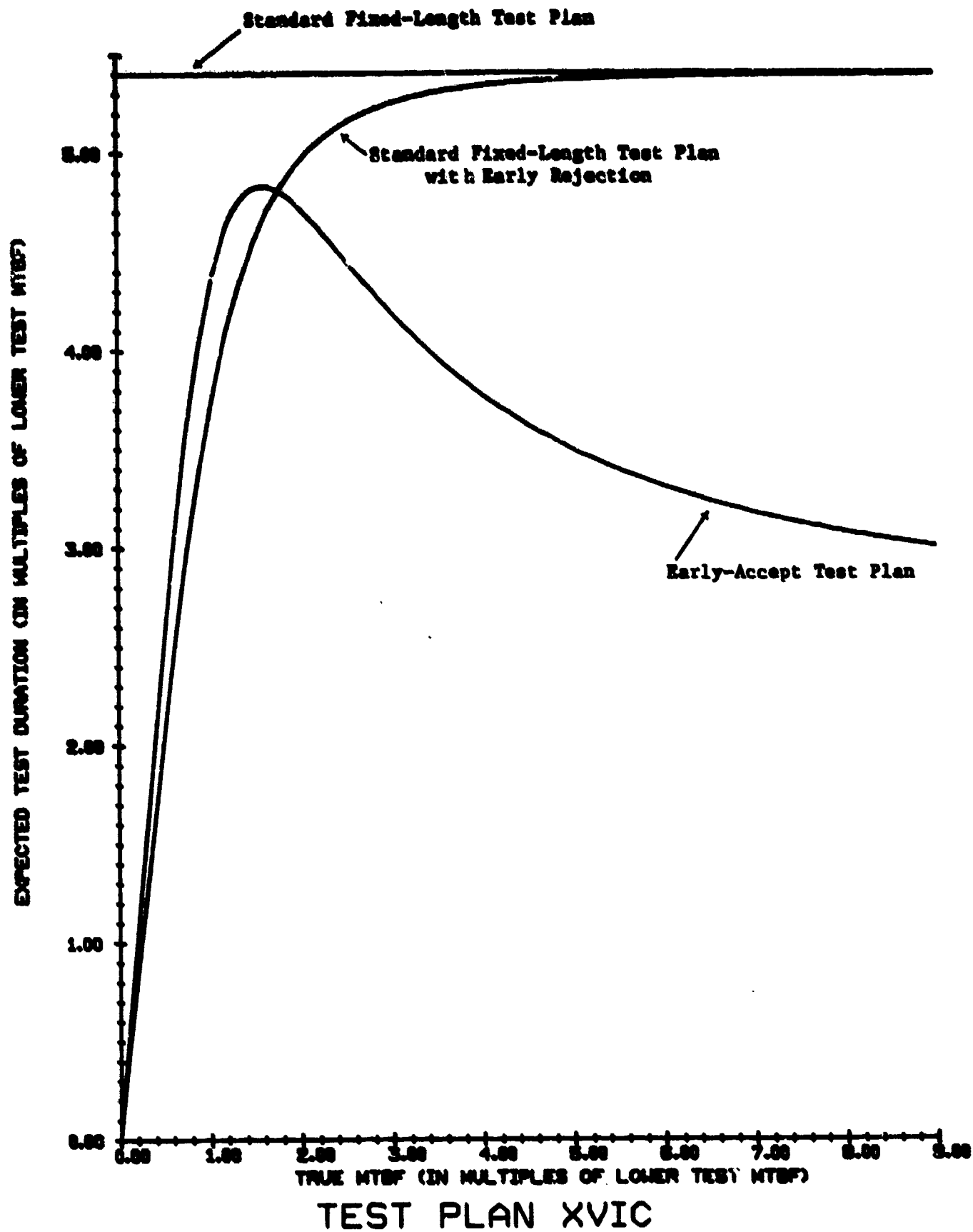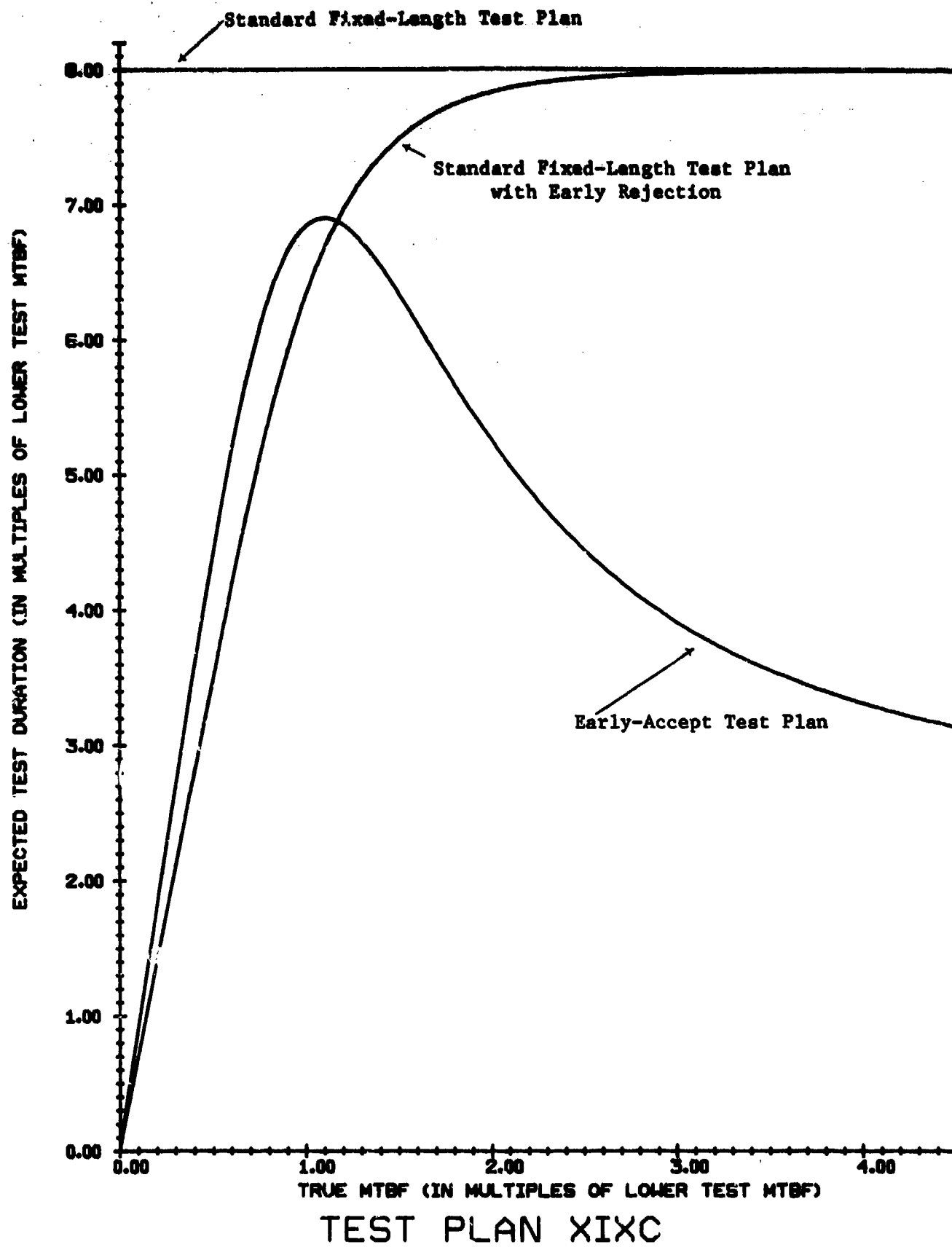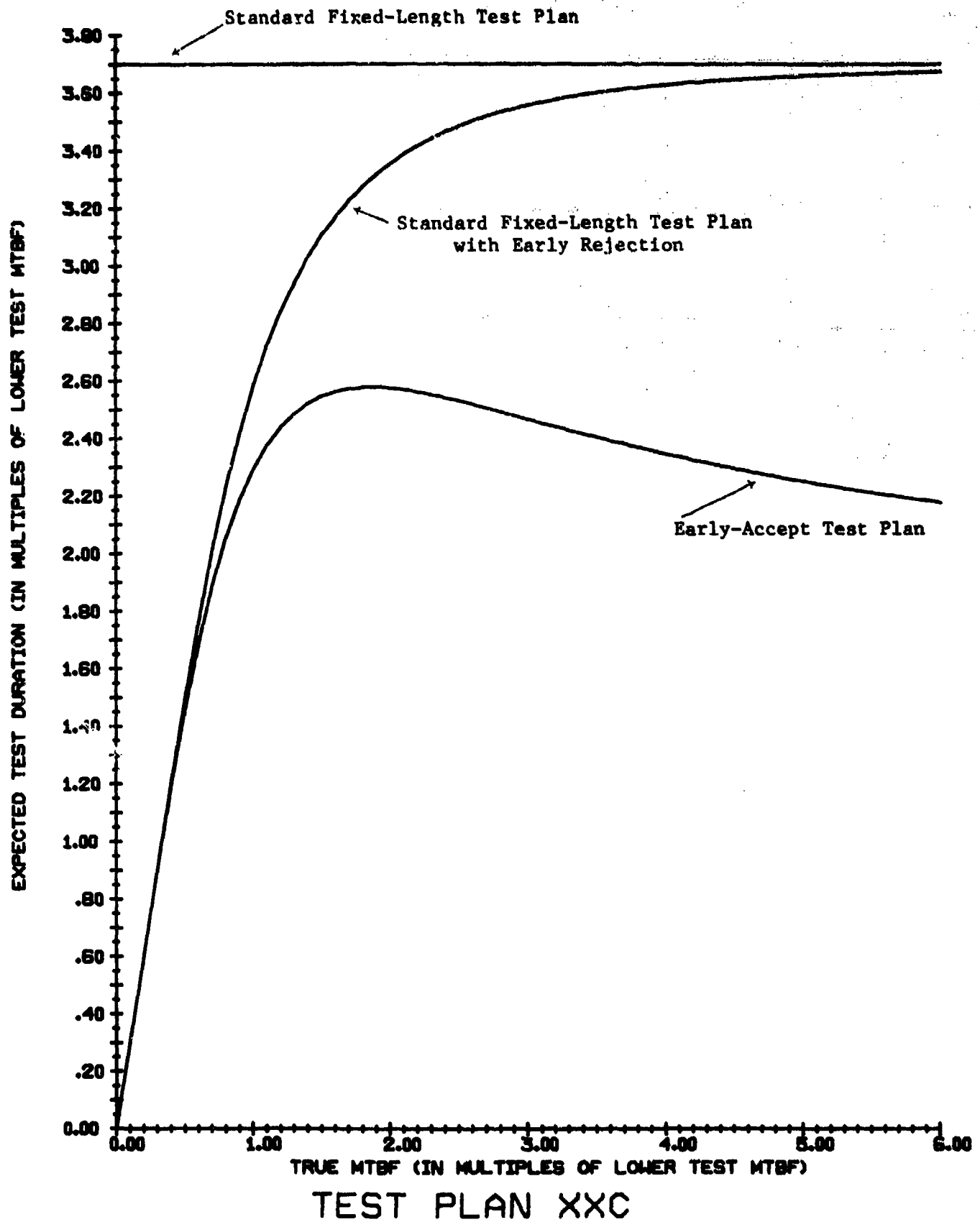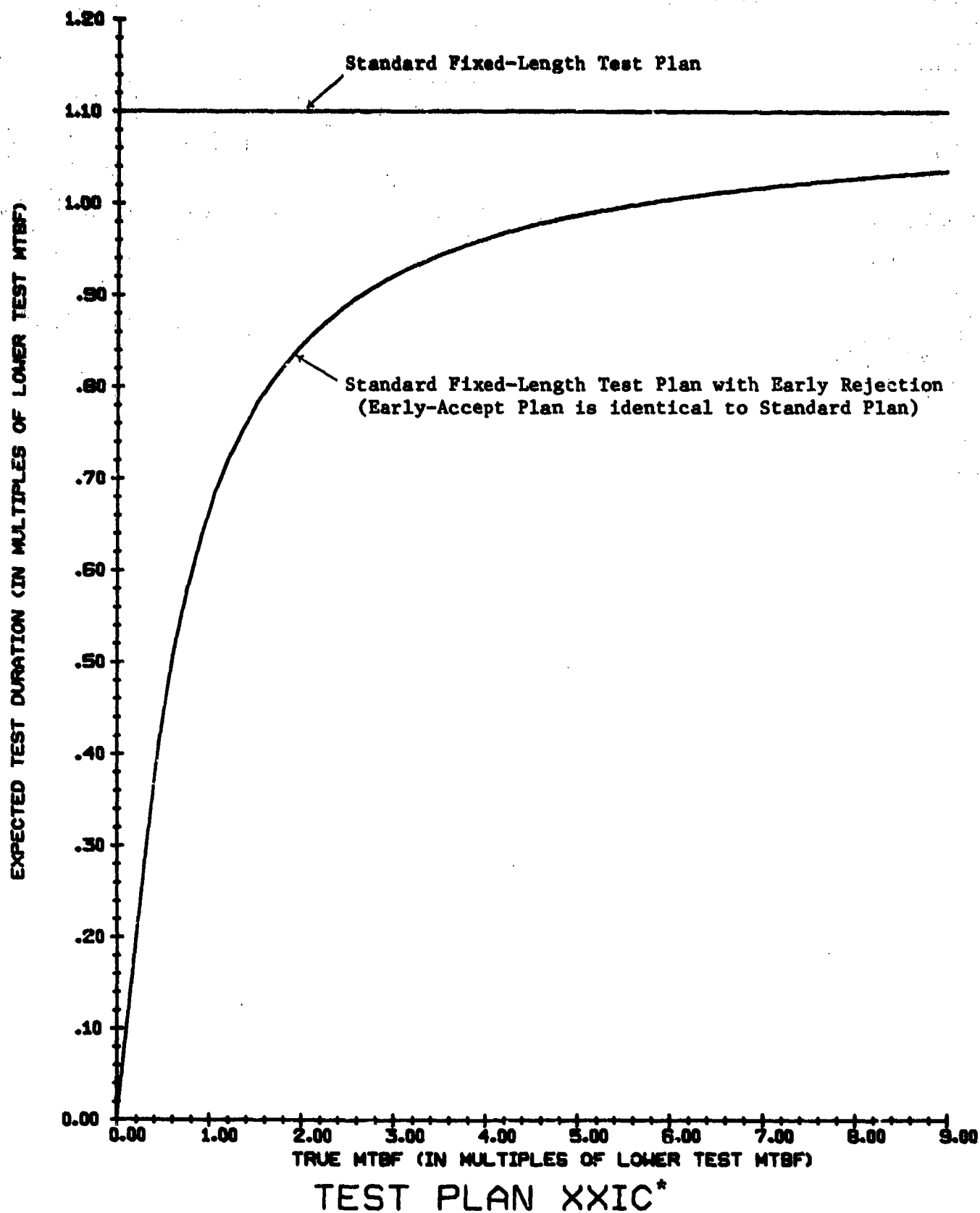
Figure 9
24

TEST PLAN XIXC

Figure 10

25

Figure 11

26

TEST PLAN XXIC*

Figure 12

*The Standard Fixed-Length Test Plan with Early Rejection and the Early-Accept
Test Plan are identical for this case.

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196 ✓	AD-A088 018	

4. TITLE (and Subtitle)	5. TYPE OF REPORT & PERIOD COVERED
AN EARLY-ACCEPT MODIFICATION TO THE TEST PLANS OF MILITARY STANDARD 781C	Technical Report
	6. PERFORMING ORG. REPORT NUMBER

7. AUTHOR(s)	8. CONTRACT OR GRANT NUMBER(s)
David A. Butler and Gerald J. Lieberman	✓ N00014-75-C-0561

9. PERFORMING ORGANIZATION NAME AND ADDRESS	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
Department of Operations Research and Department of Statistics - Stanford University, Stanford, California 94305	(NR-047-200)

11. CONTROLLING OFFICE NAME AND ADDRESS	12. REPORT DATE
Operations Research, Code 434	June 17, 1980
Office of Naval Research	13. NUMBER OF PAGES
Arlington, Virginia 22217	27

14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)	15. SECURITY CLASS. (of this report)
	Unclassified
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

APPROVED FOR PUBLIC RELEASE: DISTRIBUTION IS UNLIMITED

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 30, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

LIFE TESTING

RELIABILITY DESIGN ACCEPTANCE TESTS

RELIABILITY PRODUCTION QUALIFICATION TESTS

SEQUENTIAL TESTING

EXPONENTIAL DISTRIBUTION

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This paper is concerned with the statistical test plans contained in Military Standard 781C, "Reliability Design Qualification and Production Acceptance Tests: Exponential Distribution" and the selection and use of these plans. The Standard contains both fixed-length test plans (Plans IXC through XVIIC and XIXC through XXIC) and probability ratio sequential tests

(Cont'd)

Report No. 196 AN EARLY-ACCEPT MODIFICATION TO THE TEST PLANS OF MILITARY
                STANDARD 781C (abstract continued)

(Plans IC through VIIIC and XVIIIC). Each fixed-length test plan is
characterized by its discrimination ratio (d), its total test time (T), and its
maximum allowable number of failures to accept (k). If a fixed-length test plan
is selected, the total test duration is essentially set in advance. The only
way in which one of these plans can terminate early is by rejection. For
example, Test Plan XVIIC terminates with a reject decision at the third failure
if this failure occurs before 4.3 units of total test time have transpired.
An accept decision can only be made when 4.3 units of total test time have
accrued. Even if the second failure occurs very early, an early reject decision
cannot be made; nor can an early-accept decision be made if no failures have
occurred, say, by time 4.0. In both of these situations, an early decision
would lack statistical validity in failing to guarantee the operating charac-
teristic of the selected plan. Moreover, an early reject decision by the
consumer would probably violate contractual agreements with the producer. How-
ever, an early-accept decision by the consumer would not be subject to such an
objection. Such a decision might seem very desirable to the consumer
(government) if testing costs were substantial or if schedule deadlines were
near. This paper presents modifications to the fixed-length test plans of
MIL-STD-781C which allow early-accept decisions to be made without sacrificing
statistical validity. The proposed plans differ from the probability ratio
sequential tests in the Standard in that rejection is permitted only after a
fixed number of failures have been observed.